Relativistic heat flux for a single component charged fluid in the presence of an electromagnetic field

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Abstract

Transport properties in gases are significantly affected by temperature. In previous works it has been shown that when the thermal agitation in a gas is high enough, such that relativistic effects become relevant, heat dissipation is driven not solely by a temperature gradient but also by other vector forces [1]-[8]. In the case of relativistic charged fluids, a heat flux is driven by an electrostatic field *even in the single species case* [8]. The present work generalizes such result by considering also a magnetic field in an arbitrary inertial reference frame. The corresponding constitutive equation is explicitly obtained showing that both electric and magnetic forces contribute to thermal dissipation. This result may lead to relevant effects in plasma dynamics.

1 Introduction

Linear irreversible thermodynamics predicts linear relations between dissipative fluxes and thermodynamic forces. These constitutive, or closure, equations when generalized for high temperature fluids feature cross-like effects. In particular, it has been shown that a heat flux is driven by an external electrostatic field in a charged single component fluid. This corresponds to an electro-thermal effect in simple gases which is not present in

the non-relativistic, low temperature, case when the full Boltzmann equation is considered [8]. In the present paper such an effect is generalized to the electromagnetic case considering an additional magnetic field and working in an arbitrary reference frame.

Boltzmann's equation has been applied by other authors to address both the single component charged gas as well as the mixture in both relativistic and non-relativistic scenarios. In particular, Spitzer established a heat flux depending on the electrostatic field for a monocomponent fluid by neglecting the time derivative term, assuming a steady state situation [9, 10]. On the other hand, non-relativistic two component plasmas have been addressed widely in the literature where electro-thermal effects correspond to cross effects that rely on the system being a mixture [11, 12, 13]. Kremer et. al. adressed the relativistic charged mixture obtaining Fourier and Ohm's law as well as electro-thermal effects driven by the electric field [4].

The formalism here presented follows the standard steps in the Chapman-Enskog method whithin Marle's relaxation approximation for a charged single relativistic fluid. Once the electromagnetic contribution to the non-equilibrium part of the distribution is established, the heat flux is calculated based on the expression in terms of the energy flux arising solely from the chaotic component of the velocities [14]. The result obtained consists on a general expression for the heat flux present in a simple fluid driven by an external electromagnetic field in an arbitrary reference frame.

The rest of the paper is divided as follows. Section II is devoted to the description of the fundamental elements of relativistic kinetic theory where Boltzmann's equation is written for a charged single component fluid and the corresponding transport equations are obtained. A simplified collision model is incorporated in Sect. III in order to establish the basic structure of the non-equilibrium part of the distribution function. The heat flux is obtained in Sect. IV, where the explicit constitutive equation is shown and the corresponding transport coefficient calculated. The explicit dependence of the heat flux on the electromagnetic force is also shown. Conclusions and final remarks are included in Sect. V.

2 Transport equations for a relativistic charged simple fluid

Relativistic molecular dynamics become relevant in gases when thermal agitation is such that the particle's chaotic velocities become close to the speed of light. The relativistic nature of the gas is measured through the relativistic parameter $z = kT/mc^2$. When $z \gtrsim 1$, the thermal energy is comparable to the rest mass of the individual molecules and relativistic corrections to particle dynamics permeate, through statistical averages, to measurable corrections in state variables and their evolution equations.

From the kinetic theory point of view, the formal treatment of high temperature gases is based on the relativistic Boltzmann equation. For the case of a single charged relativistic fluid, such equation can be written as [2, 3]

$$v^{\alpha}\frac{\partial f}{\partial x^{\alpha}}+\frac{q}{m}F^{\alpha\nu}v_{\nu}\frac{\partial f}{\partial v^{\alpha}}=J\left(ff'\right) \tag{1}$$

Here f is the relativistic distribution function which, as in the non relativistic case, yields the occupation number of phase-space cells with volume d^3xd^3v. As long as the fluid is dilute and no intense gravitational fields are present, a flat Minkowski metric $\eta^{\mu\nu}$ (with a +++- signature) is appropriate where $x^{\mu}=[\vec{r},ct]$ and $v^{\mu}=\gamma_w\left[w^{\mu},c\right]$, with $\gamma_w=\left(1-\frac{w^{\ell}w_{\ell}}{c^2}\right)^{-1/2}$ being the Lorentz factor. Here and in the rest of the work the Einstein summation convention is used, with greek indices running from 1 to 4 and latin ones up to 3. Also, in Eq. (1) the external force is expressed in terms of the electromagnetic field tensor $F^{\alpha\nu}$. Notice that in the electromagnetic case, the inclusion of a velocity dependent force field does not alter the streaming side of the equation because of the antisymmetry properties of $F^{\alpha\nu}$, that is

$$\frac{\partial}{\partial v^{\alpha}}\left(F^{\alpha\nu}v_{\nu}\right)=F^{\alpha\nu}\frac{\partial v_{\nu}}{\partial v^{\alpha}}=F^{\alpha\nu}\delta_{\nu\alpha}=0 \tag{2}$$

Conserved quantities and the expressions for dissipative fluxes and state variables are obtained following a similar procedure as the one carried out in Ref. [14]. Starting from Boltzmann's equation (Eq. (1)) one multiplies both sides by a collisional invariant $\psi\left(v^{\mu}\right)$ and integrate over velocity space, which yields

$$\frac{\partial}{\partial x^{\alpha}}\int\psi\left(v^{\mu}\right)v^{\alpha}fd^*v-\int F^{\alpha\nu}v_{\nu}\psi\left(v^{\mu}\right)\frac{\partial f}{\partial v^{\alpha}}d^*v=\int J\left(ff'\right)d^*v \tag{3}$$

where the fact that velocity and position are independent variables in phase space has been used for the first term on the left hand side. The invariant volume element is given by $d^*v=\gamma_w^{-1}d^3v$ [17]. For the second term on the left hand side of Eq. (3) one has

$$\int F^{\alpha\nu}v_{\nu}\psi\left(v^{\mu}\right)\frac{\partial f}{\partial v^{\alpha}}d^*v=\int\frac{\partial}{\partial v^{\alpha}}\left[F^{\alpha\nu}v_{\nu}\psi\left(v^{\mu}\right)f\right]d^*v-\int f\psi\left(v^{\mu}\right)\frac{\partial}{\partial v^{\alpha}}\left[F^{\alpha\nu}v_{\nu}\right]d^*v-\int fF^{\alpha\nu}v_{\nu}\frac{\partial}{\partial v^{\alpha}}\left[\psi\left(v^{\mu}\right)\right]d^*v \tag{4}$$

where the first term vanishes using Gauss' theorem and the fact that the distribition function vanishes (exponentially) on the boundaries of the v domain. The second term also vanishes because of the antisymmetry of the field tensor. For the right hand side, the symmetry properties of the collision operator, together with

the fact that $\psi\left(v^{\mu}\right)$ is a collisional invariant, leads to the vanishing of such term and thus one obtains

$$\frac{\partial}{\partial x^{\alpha}}\int\psi\left(v^{\alpha}\right)v^{\alpha}fd^{*}v-\frac{q}{m}\int fF^{\alpha\nu}v_{\nu}\frac{\partial\psi\left(v^{\beta}\right)}{\partial v^{\alpha}}d^{*}v=0 \tag{5}$$

Considering $\psi\left(v^{\mu}\right)=1$ in Eq. (5) one has

$$\frac{\partial N^{\alpha}}{\partial x^{\alpha}}=0 \tag{6}$$

with $N^{\alpha}=\int v^{\alpha}fd^{*}v$ being the particle four flux. The relation of this flux with the hydrodynamic four velocity $\mathcal{U}^{\alpha}$ follows as in the neutral case as

$$N^{\alpha}=n\mathcal{U}^{\alpha} \tag{7}$$

and thus, the conservation of such quantity leads to the continuity equation namely,

$$\mathcal{U}^{\mu}\frac{\partial n}{\partial x^{\mu}}=-n\frac{\partial\mathcal{U}^{\mu}}{\partial x^{\mu}} \tag{8}$$

For the momentum-energy invariant $\psi\left(v^{\beta}\right)=mv^{\beta}$ one obtains the following balance equation

$$\frac{\partial T^{\beta\alpha}}{\partial x^{\alpha}}=qn\mathcal{U}_{\nu}F^{\beta\nu} \tag{9}$$

where the fluid's energy-momentum tensor is defined as

$$T^{\alpha\beta}=\int mv^{\alpha}v^{\beta}fd^{*}v \tag{10}$$

Also, by expressing this quantity in terms of a 3+1 decomposition of a symmetric second rank tensor leads to the same expression as in Ref. [14], that is

$$T^{\alpha\beta}=\frac{n\varepsilon}{c^{2}}\mathcal{U}^{\alpha}\mathcal{U}^{\beta}+ph^{\alpha\beta}+\frac{1}{c^{2}}q^{\alpha}\mathcal{U}^{\beta}+\frac{1}{c^{2}}q^{\beta}\mathcal{U}^{\alpha}+\Pi^{\alpha\beta} \tag{11}$$

where ε is the internal energy density, p the hydrostatic pressure, q^{α} the heat flux and $\Pi^{\alpha\beta}$ Navier's viscous tensor. Also, $h^{\alpha\beta}$ is the so-called spatial projector which is given by

$$h^{\mu\nu}=\eta^{\mu\nu}+\frac{\mathcal{U}^{\mu}\mathcal{U}^{\nu}}{c^{2}} \tag{12}$$

The force term on the right hand side of Eq. (9) can be expressed as the four-divergence of the electromagnetic

stress tensor $M^{\alpha\beta}$ [3, 15] such that the energy-momentum conservation equation for the single charged fluid can be written as

$$\frac{\partial}{\partial x^{\alpha}}\left(T^{\beta\alpha}+M^{\alpha\beta}\right)=0 \tag{13}$$

Equation (13) contains both the momentum balance equation and the total energy balance. In order to write an expression for the internal energy evolution one projects Eq. (25) in the direction of the hydrodynamic velocity namely

$$\mathcal{U}_{\beta}\frac{\partial}{\partial x^{\alpha}}\left(T^{\beta\alpha}+M^{\alpha\beta}\right)=0 \tag{14}$$

which yields

$$n\dot{\varepsilon}+p\frac{\partial\mathcal{U}^{\mu}}{\partial x^{\nu}}=\mathcal{U}_{\mu}\frac{1}{c^{2}}\dot{q}^{\mu}-\frac{\partial q^{\nu}}{\partial x^{\nu}} \tag{15}$$

where the dot indicates a fluid's proper time derivative, i.e. $\dot{A}^{\mu}=\mathcal{U}^{\nu}\frac{\partial A^{\mu}}{\partial x^{\nu}}$. Notice that the electromagnetic field term vanishes since

$$\mathcal{U}_{\mu}\frac{\partial M^{\mu\nu}}{\partial x^{\nu}}=qF^{\mu\nu}\mathcal{U}_{\mu}\mathcal{U}_{\nu}=0 \tag{16}$$

where we have used that $F^{\mu\nu}$ is antisymmetric. For the momentum balance one obtains, from Eq. (13),

$$\left(\frac{n\varepsilon}{c^{2}}+\frac{p}{c^{2}}\right)\dot{\mathcal{U}}^{\mu}+\left(p_{,\alpha}+\frac{1}{c^{2}}\dot{q}_{\alpha}+\pi^{\nu}_{\alpha;\nu}\right)h^{\mu\alpha}+\frac{1}{c^{2}}\left(\mathcal{U}^{\mu}_{;\nu}q^{\nu}+\theta q^{\mu}\right)=qF^{\mu\nu}\mathcal{U}_{\nu} \tag{17}$$

Equations (8), (15) and (17) constitute the set of transport equations for a relativistic single component inviscid fluid in the presence of an electromagnetic field . A constitutive equation that expresses the heat flux in terms of the forces is required in order to close the set such that it can appropriately describe the corresponding dynamics. The Chapman-Enskog procedure will be used in the following sections in order to obtain such a relation.

3 The electromagnetic contribution to $f^{(1)}$

In this section, the Chapman-Enskog method is used in order to establish a first order in the gradients correction to the equilibrium distribution function using Eq. (1). For the system here considered, all gradients of the hydrodynamic velocity are negelcted since the heat flux, being a vector flux, will not be coupled to second rank tensor forces.

For the system in hand, a simple charged gas in the presence of an electromagnetic field, the acceleration acting on the molecules that appears in the second term of Eq. (**??**) is calculated using the electromagnetic

field tensor which, in cartesian coordinates reads

$$F^{\mu\nu}=\begin{pmatrix} 0 & B_z & -B_y & -\frac{E_x}{c} \\ -B_z & 0 & B_x & -\frac{E_y}{c} \\ B_y & -B_x & 0 & -\frac{E_z}{c} \\ \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} & 0 \end{pmatrix} \tag{18}$$

Also, for the sake of simplicity, a relaxation time model will be introduced on the right hand side of Boltzmann's equation by assuming that all details of collisions can be included in one parameter τ such that

$$J\left(ff'\right)=-\frac{f-f^{(0)}}{\tau} \tag{19}$$

This simple model has been extensively used in order to assess the structure of the fluxes-forces relations for single component fluids. For a more precise value of the transport coefficients present in such relations, the use of the complete kernel is required. Introducing Eqs. (19) and (18) in Eq. (1) together with Chapman-Enskog's hypothesis, namely

$$f=f^{(0)}+f^{(1)} \tag{20}$$

one obtains for the non-equilibrium correction to the distribution function

$$f^{(1)}=-\tau\left\{v^{\alpha}f^{(0)}_{,\alpha}+\left(\frac{q}{m}v_{\nu}F^{\mu\nu}\right)\frac{\partial f^{(0)}}{\partial v^{\mu}}\right\} \tag{21}$$

where

$$f^{(0)}=\frac{n}{4\pi c^{3}zK_{2}\left(\frac{1}{z}\right)}e^{\frac{\mathcal{U}_{\beta}v^{\beta}}{zc^{2}}} \tag{22}$$

is the local equilibrium solution [2] which leads to the expression [8]

$$f^{(1)}=-\tau\left\{v^{\alpha}f^{(0)}_{,\alpha}+\left(\frac{q}{mc^{2}z}F^{\mu\nu}v_{\nu}\mathcal{U}_{\mu}\right)f^{(0)}\right\} \tag{23}$$

In Eq. (23) $K_n\left(\frac{1}{z}\right)$ is the $n-$th order modified Bessel function of the second kind. On the other hand, for the first term on the right hand side of Eq. (21), the local equilibrium Euler equations need to be introduced in order to write the time derivatives of the state variables, that arise from the functional hypothesis $f\left(x^{\nu},v^{\nu}\right)=f\left(v^{\nu},n\left(x^{\nu}\right),T\left(x^{\nu}\right),\mathcal{U}\left(x^{\nu}\right)\right)$, in terms of the forces. In order to separate time and space derivatives

in such a term we introduce the following decomposition for an arbitrary reference frame

$$v^{\alpha} = v^{\beta} h^{\alpha}_{\ \beta} - \left(\frac{v^{\beta} \mathcal{U}_{\beta}}{c^2} \right) \mathcal{U}^{\alpha} \tag{24}$$

such that

$$v^{\alpha} f^{(0)}_{,\alpha} = v^{\beta} h^{\alpha}_{\ \beta} f^{(0)}_{,\alpha} - \left(\frac{v^{\beta} \mathcal{U}_{\beta}}{c^2} \right) \mathcal{U}^{\alpha} f^{(0)}_{,\alpha} \tag{25}$$

For the first term in Eq. (39) one has

$$v^{\beta} h^{\alpha}_{\ \beta} f^{(0)}_{,\alpha} = v^{\beta} h^{\alpha}_{\ \beta} f^{(0)} \left[\frac{n_{,\alpha}}{n} + \frac{T_{,\alpha}}{T} \left(1 + \frac{v^{\beta} \mathcal{U}_{\beta}}{z c^2} - \frac{\mathcal{G}\left(\frac{1}{z}\right)}{z} \right) + \frac{v_{\beta}}{z c^2} \mathcal{U}^{\beta}_{\ ,\alpha} \right] \tag{26}$$

where $\mathcal{G}\left(\frac{1}{z}\right) = K_3\left(\frac{1}{z}\right) / K_2\left(\frac{1}{z}\right)$ and for the second one

$$\mathcal{U}^{\alpha} f^{(0)}_{,\alpha} = f^{(0)} \left[\frac{1}{n} \mathcal{U}^{\alpha} n_{,\alpha} + \frac{1}{T} \left(1 + \frac{v^{\beta} \mathcal{U}_{\beta}}{z c^2} - \frac{\mathcal{G}\left(\frac{1}{z}\right)}{z} \right) \mathcal{U}^{\alpha} T_{,\alpha} + \frac{v_{\beta}}{z c^2} \mathcal{U}^{\alpha} \mathcal{U}^{\beta}_{\ ,\alpha} \right] \tag{27}$$

As mentioned above, the proper time derivatives are eliminated intriducing Euler's equations which are obtained by neglecting dissipative terms in the transport equations (8), (15) and (17), which are given by

$$\mathcal{U}^{\mu} \frac{\partial n}{\partial x^{\mu}} = -n \frac{\partial \mathcal{U}^{\mu}}{\partial x^{\mu}} \tag{28}$$

$$\left(\frac{n \varepsilon}{c^2} + \frac{p}{c^2} \right) \dot{\mathcal{U}}^{\mu} + p_{,\alpha} h^{\mu\alpha} = q F^{\mu\nu} \mathcal{U}_{\nu} \tag{29}$$

$$\mathcal{U}^{\nu} T_{,\nu} = -\frac{p}{n C_n} \mathcal{U}^{\nu}_{,\nu} \tag{30}$$

where the specific heat at constant number density $C_n = \left(\frac{\partial \varepsilon}{\partial T}\right)_n$ has been introduced. Introducing Eqs. (26-30) in Eq. (39) yields the following expression

$$\begin{aligned} v^{\alpha} f^{(0)}_{,\alpha} = f^{(0)} \Bigg\{ & v^{\beta} h^{\alpha}_{\beta} \left[\frac{n_{,\alpha}}{n} + \frac{T_{,\alpha}}{T} \left(1 + \frac{v^{\beta} \mathcal{U}_{\beta}}{z c^2} - \frac{\mathcal{G}\left(\frac{1}{z}\right)}{z} \right) + \frac{v_{\mu}}{z c^2} \mathcal{U}^{\mu}_{,\alpha} \right] \\ & - \left(\frac{v^{\beta} \mathcal{U}_{\beta}}{c^2} \right) \left[-\mathcal{U}^{\mu}_{,\mu} - \frac{p}{n T C_n} \mathcal{U}^{\nu}_{,\nu} \left(1 + \frac{v^{\beta} \mathcal{U}_{\beta}}{z c^2} - \frac{\mathcal{G}\left(\frac{1}{z}\right)}{z} \right) + \frac{v_{\mu}}{\tilde{\rho} z c^2} \left(-p_{,\nu} h^{\mu\nu} + n q F^{\mu\nu} \mathcal{U}_{\nu} \right) \right] \Bigg\} \end{aligned} \tag{31}$$

Notice that the force term in Eq. (31) will compete with the second term on the right hand side of Eq. (23). Isolating these two electromagnetic contributions to the deviation from equilibrium of the distribution

function due to them, which we call $f^{(1)}_{EM}$, we obtain

$$f^{(1)}_{EM} = \frac{\tau q}{zmc^2} F^{\mu\nu} v_\mu \mathcal{U}_\nu f^{(0)} \left\{ \left(\frac{v^\beta \mathcal{U}_\beta}{c^2} \right) \frac{1}{\mathcal{G}\left(\frac{1}{z}\right)} + 1 \right\} \tag{32}$$

where we have used $\tilde{\rho} = nm\mathcal{G}\left(\frac{1}{z}\right)$. Equation (32) is the main result of this section. It consitutes the generalization of the result obtained in Ref. [8] where the fact that a purely electrostatic field can drive a single charged gas out of equilibrium in a relativistic scenario. In the present work, the generalization is twofold in the sense that it includes a magnetic field and is consequently expressed in an arbitrary reference frame. This expression will be used in the next section in order to establish the heat flux due to this effect.

4 Calculation of the heat flux

As extensively discussed in Refs. [14] and [19], heat is defined as the transport of internal energy arising from the chaotic motion of the molecules which is in principle a quantity characteristic of the comoving frame in which all mechanical effects are absent. Also, as shown in Ref. [14], it can be constructied as a tensor quantity and can thus be calculated in any arbitrary frame. The expression

$$Q^\beta = h^\beta_\eta \mathcal{L}^\eta_\gamma \int k^\gamma f^{(1)} \gamma^2_{(k)} d^* K \tag{33}$$

is here used in order to assess the contribution to thermal dissipation due to the electromagnetic field [14] in an arbitrary reference frame in which the hydrodynamic four velocity of the fluid is given by $\mathcal{U}^\nu$. In Eq. (33) $\mathcal{L}^\eta_\gamma$ is a Lorentz boost with velocity $\vec{u}$ and $K^\nu = \gamma_{(k)} \left[\vec{k}, c\right]$ is the chaotic velocity, that is the particles' velocities measured by an observer comoving with the fluid's volume element .

In order to calculate the contribution to the integral in Eq. (33) arising from $f^{(1)}_{EM}$, it needs to be written in terms of the chaotic velocity. In order to accomplish such task, we use the invariant expression [19]

$$\frac{v^\beta \mathcal{U}_\beta}{c^2} = -\gamma_{(k)} \tag{34}$$

and also write the molecular velocity as

$$v_\mu = \mathcal{L}_{\mu\nu} K^\nu = \gamma_{(k)} \mathcal{L}_{\mu\nu} k^\nu \tag{35}$$

Introducing these expressions in Eq. (32) one obtains

$$f_{EM}^{(1)} = -\frac{\tau q}{zmc^2}\mathcal{L}_{\mu\alpha}F^{\mu\nu}\mathcal{U}_\nu\gamma_{(k)}k^\alpha\left\{\frac{\gamma_{(k)}}{\mathcal{G}\left(\frac{1}{z}\right)}-1\right\}f^{(0)} \tag{36}$$

Thus, the electromagnetic contribution to the heat flux is given by

$$Q_{EM}^\beta = -\frac{\tau q}{z}\mathcal{L}_{\mu\alpha}F^{\mu\nu}\mathcal{U}_\nu h_\eta^\beta\mathcal{L}_\gamma^\eta\int f^{(0)}k^\alpha k^\gamma\left\{\frac{\gamma_{(k)}}{\mathcal{G}\left(\frac{1}{z}\right)}-1\right\}\gamma_{(k)}^3 d^*K \tag{37}$$

which can also be written as

$$Q_{EM}^\beta = -\frac{\tau q}{z}\mathcal{L}_{\mu\alpha}\mathcal{L}_\gamma^\eta h_\eta^\beta F^{\mu\nu}\mathcal{U}_\nu\mathcal{I}^{\gamma\alpha}\left(z\right) \tag{38}$$

where the integral $\mathcal{I}^{\alpha\gamma}$ is given by

$$\mathcal{I}^{\alpha\gamma}\left(z\right) = 4\pi c^3\int f^{(0)}k^\alpha k^\gamma\left\{\frac{\gamma_{(k)}}{\mathcal{G}\left(\frac{1}{z}\right)}-1\right\}\gamma_{(k)}^3\sqrt{\gamma_{(k)}^2-1}d\gamma \tag{39}$$

Equation (38) contains the main result of this work: the heat flux for a single component charged fluid in the special relativistic regime features a contribution due to the electromagnetic force $qF^{\mu\nu}\mathcal{U}_\nu$. This effect is purely relativistic and generalices the result obtained in Ref. [8] where the particular case of a purely electrostatic field in a comoving frame was addressed. It is important to emphasize that the dissipative flux in Eq. (38) is a tensor quantity.

In order to assess the magnitude of the corresponding transport coefficient, the values for the integral $\mathcal{I}^{ab}\left(z\right)$ need to be obtained. This can be readily done yielding

$$\mathcal{I}^{ab}\left(z\right) = 0, \quad \text{for } a \neq b,\ a,b = 1,2,3,4$$

$$\mathcal{I}^{aa}\left(z\right) = nzc^2\left(5z+\frac{1}{\mathcal{G}\left(\frac{1}{z}\right)}-\mathcal{G}\left(\frac{1}{z}\right)\right) \quad \text{for } a = 1,2,3$$

$$\mathcal{I}^{44}\left(z\right) = nzc^2\left(5z+\frac{2}{3}\frac{1}{\mathcal{G}\left(\frac{1}{z}\right)}-\mathcal{G}\left(\frac{1}{z}\right)\right)$$

Also notice that the $\gamma = \alpha = 4$ term in Eq. (38) vanishes since $\mathcal{L}_{\mu 4} \propto \mathcal{U}_\mu$ and thus one can write

$$Q_{EM}^\beta = -\kappa_{NR}\left(\frac{q}{kT}\right)\frac{2}{5}\left(5+\frac{1}{z\mathcal{G}\left(\frac{1}{z}\right)}-\frac{\mathcal{G}\left(\frac{1}{z}\right)}{z}\right)\mathcal{L}_{\mu b}\mathcal{L}_a^\eta h_\eta^\beta F^{\mu\nu}\mathcal{U}_\nu\delta^{ab} \tag{40}$$

with $a,b = 1,2,3$. The non-relativistic thermal conductivity in the relaxation time approximation $\kappa_{NR} =$

$\frac{5}{2}\frac{nk^2T^2}{m}\tau$ has been here introduced.

The components of the heat flux in Eq. (40) can be explicitly obtained in an arbitrary frame where $\mathcal{U}^\nu = [\vec{u}, c]$, which yieds:

$$Q^\nu_{EM} = -\kappa_{NR}\left(\frac{q}{kT}\right)\frac{2}{5}\left(5 + \frac{1}{z\mathcal{G}\left(\frac{1}{z}\right)} - \frac{\mathcal{G}\left(\frac{1}{z}\right)}{z}\right)\gamma_u\left[\vec{E} + \vec{u}\times\vec{B}, \vec{E}\cdot\frac{\vec{u}}{c}\right] \tag{41}$$

from which one can extract three relevant limits. Firstly notice that in the comoving frame where $\mathcal{U}^\nu = \left[\vec{0}, c\right]$ and $\gamma_u = 1$ one obtains

$$Q^\nu_{EM} = -\kappa_{NR}\left(\frac{q}{kT}\right)\frac{2}{5}\left(5 + \frac{1}{z\mathcal{G}\left(\frac{1}{z}\right)} - \frac{\mathcal{G}\left(\frac{1}{z}\right)}{z}\right)\left[\vec{E}, 0\right]$$

which is precisely the result obtained in Ref. [8] for the case of a purely electrostatic field in the fluid's comoving frame. The other two limits pertain the non-relativistic case. For a fluid with a high value of z but such that the hydrodynamic velocity is considerably lower than the speed of light, $\gamma_u \sim 1$ and

$$Q^\nu_{EM} \sim -\kappa_{NR}\left(\frac{q}{kT}\right)\frac{2}{5}\left(5 + \frac{1}{z\mathcal{G}\left(\frac{1}{z}\right)} - \frac{\mathcal{G}\left(\frac{1}{z}\right)}{z}\right)\left[\left(\vec{E} + \vec{u}\times\vec{B}\right), \vec{E}\cdot\frac{\vec{u}}{c}\right] \tag{42}$$

which is physically meaningfull since the thermodynamic force in this limit is the electromagnetic force for the spatial components and corresponds to the mechanical electrostatic dissipation on the fourth component. Finally, the striclty non-relativistic limit in which also the temperature is low such that $z \ll 1$ one has

$$Q^\nu_{EM} \sim -\kappa_{NR}\left(\frac{q}{kT}\right)\left(z - z^2 + ...\right)\left[\left(\vec{E} + \vec{u}\times\vec{B}\right), \vec{E}\cdot\frac{\vec{u}}{c}\right] \tag{43}$$

finally showing that this effect is strictly relativistic.

5 Final remarks

This paper has been devoted to the extension of the Benedicks-type effect recently identified in the context of special relativity to the magnetic field case. Although the calculation may seem to be a straightforward exercise considering the inclusion of the magnetic field components in Faraday's field tensor when calculating $f^{(1)}$, there are several features that must be highlighted. The fact that $\vec{B}$ appears explicitly in $f^{(1)}$ implies a non-zero contribution of the magnetic field to the entropy production present in a single component fluid. In contrast, in the non-relativistic case the magnetic field does not contribute to the entropy production at the Navier-Stokes level [18]. More over the hydrodynamic equations now include new terms in which the field

is now identified as a thermodynamic force. This is rather new, since the only dissipative effects related to magnetic fields previously identified at first order in the Knudsen parameter (for simple fluids) were those associated to the values of the viscosity coefficients [18, 12, 13].

Magnetic fields are well known to be ubiquous in the universe. Although the corresponding astrophysical mean free times are quite large compared to the ordinary scales, the characteristic times for most processes are such that dissipative effects must be taken into account [Spitzer]. It is in this type of scenarios in which irreversible thermodynamics becomes irrelevant. Future work will be devoted the study of hydromagnetic instabilities including the effects presented in this work.

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